Lorentz-invariant look at quantum clock-synchronization protocols based on distributed entanglement

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Recent work has raised the possibility that quantum-information-theory techniques can be used to synchronize atomic clocks nonlocally. One of the proposed algorithms for quantum clock synchronization (QCS) requires distribution of entangled pure singlets to the synchronizing parties [R. Jozsa *et al.*, Phys. Rev. Lett. **85** 2010 (2000)]. Such remote entanglement distribution normally creates a relative phase error in the distributed singlet state, which then needs to be purified asynchronously. We present a relativistic analysis of the QCS protocol that shows that asynchronous entanglement purification is not possible, and, therefore, the proposed QCS scheme remains incomplete. We discuss possible directions of research in quantum-information theory, which may lead to a complete, working QCS protocol.

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I. INTRODUCTION: A QUANTUM PROTOCOL FOR CLOCK SYNCHRONIZATION

Suppose a supply of identical but distinguishable two-state systems (e.g., atoms) is available whose between-state transitions can be manipulated (e.g., by laser pulses). Let $|1\rangle$ and $|0\rangle$ denote, respectively, the excited and ground states of the prototype two-state system (which span the internal Hilbert space $\mathcal{H}$), and let the energy difference between the two states be Ω (we will use units in which $\hbar=c=1$ throughout this paper). Without loss of generality, we can assume

$$\hat{H}_0|0\rangle=0, \quad \hat{H}_0|1\rangle=\Omega|1\rangle, \tag{1}$$

where $\hat{H}_0$ denotes the internal Hamiltonian operator. Suppose pairs of these two-state systems are distributed to two spatially separated observers Alice and Bob. The Hilbert space of each pair can be written as $\mathcal{H}_A\otimes\mathcal{H}_B$, where $\otimes$ denotes the tensor product of the two vector spaces. A (''perfect'') singlet is the specific entangled quantum state in this product Hilbert space given by

$$\Psi=\frac{1}{\sqrt{2}}(|0\rangle_A\otimes|1\rangle_B-|1\rangle_A\otimes|0\rangle_B). \tag{2}$$

[In what follows, we will omit tensor-product signs in expressions of the kind of Eq. (2) unless required for clarity.] Two important properties of the singlet state Ψ are as follows. (i) It is a ''dark'' state (invariant up to a multiplicative phase factor) under the time evolution $\hat{U}_t\equiv\exp(it\hat{H}_0)$, i.e., $(\hat{U}_t\otimes\hat{U}_t)\Psi=e^{i\phi}\Psi$, where $e^{i\phi}$ is an overall phase, and (ii) it is similarly invariant under all unitary transformations of the form $\hat{U}\otimes\hat{U}$, where $\hat{U}$ is any arbitrary unitary map on $\mathcal{H}$ (not necessarily equal to $\hat{U}_t$). Both properties are needed for the quantum clock-synchronization (QCS) protocol of Jozsa *et al.* [1], which assumes a supply of such pure singlet states shared as a resource between the synchronizing parties Alice and Bob (in addition, Bob and Alice are assumed to be stationary with respect to a common reference frame). Specifically, consider the unitary (Hadamard) transformation ($\pi/2$ pulse followed by the spin operator $\hat{\sigma}_z$) on $\mathcal{H}$ given by

$$|0\rangle\mapsto|+\rangle\equiv\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle),$$

$$|1\rangle\mapsto|-\rangle\equiv\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle). \tag{3}$$

Unlike the states $|0\rangle$ and $|1\rangle$, which are dark under time evolution (they only pick up an overall phase under $\hat{U}_t$), the states $|+\rangle$ and $|-\rangle$ are ''clock states'' (in other words, they accumulate an observable relative phase under $\hat{U}_t$) because of the energy difference Ω as specified in Eq. (1). Such states can be used to ''drive'' precision clocks in the following way: Start, for example, with an ensemble of atoms in the state $|+\rangle$ produced by an initial Hadamard pulse at time t_0, and apply a second Hadamard pulse at a later time t_0+T. This leads to a final state at t_0+T equivalent, up to an overall phase factor, to the state

$$\cos\left(\frac{\Omega}{2}T\right)|0\rangle+i\sin\left(\frac{\Omega}{2}T\right)|1\rangle. \tag{4}$$

Measurement of the statistics (relative populations) of ground vs excited atoms in the state Eq. (4) then yields a precision measurement of the time interval T; hence clock functionality for $|+\rangle$. [In practice, such measurements are used to stabilize the frequency of a relatively noisy local oscillator (typically a maser), whose (stabilized) oscillations then drive the ultimate clock readout.] Now, the invariance of the pure singlet Ψ [Eq. (2)] under the Hadamard transformation Eq. (3) can be seen explicitly in the alternative representation

$$\Psi = \frac{1}{\sqrt{2}}(|-\rangle_A \otimes |+\rangle_B - |+\rangle_A \otimes |-\rangle_B). \tag{5}$$

Here, in Eq. (5), we have the crux of the QCS algorithm of Ref. [1]: The dark, invariant state Ψ, shared between Alice and Bob, contains two clock states, one for each observer, entangled in such a way as to ''freeze'' their time evolution. As soon as Bob or Alice performs a measurement on Ψ in the basis $\{|+\rangle,|-\rangle\}$, thereby destroying the entanglement, he or she starts these two dormant clocks ''simultaneously'' in both reference frames. Classical communications are then necessary to sort out which party has the $|+\rangle$ clock and which party has $|-\rangle$. When used to stabilize identical quantum clocks at each party's location, these correlated clock states then provide precise time synchrony between Bob and Alice [2].

It is important to emphasize that the nondegenerate nature of the singlet state Ψ [Eq. (2)] is crucial for the QCS protocol to work. This is in complete contrast with other quantum-information-theory protocols (such as teleportation [3], quantum cryptographic key distribution [4], and others) all of which will work equally well with degenerate ($\Omega = 0$) singlets.

What is the significance of entanglement in the above protocol? As has been pointed out by a number of authors [5,6] following the original publication [1], the QCS protocol is completely equivalent to slow clock transport as long as the entanglement in the singlet state Eq. (2) is distributed by transporting the entangled pairs kinematically to the synchronizing parties Alice and Bob (see also the discussion in Sec. II below). The potentially far-reaching consequences of the QCS algorithm become clear when we realize that the physical transport of entangled constituents is by no means the only way to distribute entanglement, though it is by far the most obvious.

Notice that *provided* such a ''nonlocal'' method of entanglement distribution is available to practically create pure singlets of the form Eq. (2), the QCS algorithm gives a way of synchronizing clocks across arbitrarily large distances, independent of the medium that separates the two atomic clocks to be synchronized—so long as a classical communications link exists between the two synchronizing parties. Since the synchrony transfer takes place instantaneously over the quantum channel, no timing information needs to be passed over the classical channel. This allows the protocol to bypass a number of noise sources present on the classical link (such as an interceding medium with fluctuating index of refraction), which currently limit the accuracy of satellite-to-satellite and satellite-to-ground synchronization protocols.

There are a number of ''nonlocal'' entanglement transfer protocols that have been discussed in the theory literature, and some of these are briefly considered in Sec. VI below. Most of the rest of this paper, however, is devoted to the analysis of what is perhaps the next most obvious method of entanglement transfer: entanglement purification. The idea of entanglement purification is to distribute the entangled state, Eq. (2), to the synchronizing parties in some noisy manner (possibly via simple kinematical transport), and then to purify the resulting imperfect singlet state by using some sort of asynchronous purification protocol (i.e., one that does not rely on preestablished time synchrony between the local clocks of Alice and Bob) which may involve (asynchronous) classical communication between the parties (as well as the loss of some fraction of the noisy singlets depending on the fidelity of the original transport and the yield of the purification protocol). In this paper we will give an answer to the fundamental question: Is asynchronous entanglement purification possible?

II. THE PRESKILL PHASE OFFSET

In principle, the QCS protocol as outlined in Sec. I is rigorously correct and self-contained. If our Universe somehow possessed primordial nondegenerate singlet states Ψ (leftover as ''relics'' from the Big Bang), the protocol just described would be perfectly sufficient to implement ultraprecise clock synchronization between comoving distant observers. In practice, however, the QCS algorithm can reasonably be viewed as simply reducing the problem of clock synchronization to the problem of distributing pure entanglement to spatially separated regions of space-time. To see that the latter is a nontrivial problem, consider the simplest way one would attempt to distribute entanglement to remote regions: start with locally created pairs of two-level systems (atoms) in pure singlet states Ψ of the form Eq. (2), and transport the two subsystems separately to the locations of Bob and Alice. The internal Hamiltonians of the two subsystems while in transport can be written in the form

$$\hat{H}_A = \hat{H}_0 + \hat{H}_A{}^{\text{ext}}, \quad \hat{H}_B = \hat{H}_0 + \hat{H}_B{}^{\text{ext}}, \tag{6}$$

where $\hat{H}_A{}^{\text{ext}}$ and $\hat{H}_B{}^{\text{ext}}$ denote interaction Hamiltonians arising from the coupling of each subsystem to its external environment, and, unless the environment, which each subsystem is subject to during transport is precisely controlled, $\hat{H}_A{}^{\text{ext}} \neq \hat{H}_B{}^{\text{ext}}$ in general, leading to a relative phase offset in the final entangled state. Furthermore, unless the world lines of the transported subsystems are arranged to have precisely the same Lorentz length (proper time), a further contribution to this phase offset would occur due to the proper-time delay between the two world-lines (see also the discussion in Sec. IV below). The end result is an imperfect singlet state

$$\Psi_\delta = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - e^{i\delta}|1\rangle_A|0\rangle_B), \tag{7}$$

where δ is a real phase offset that is fixed but entirely unknown, which we call ''the Preskill phase'' in honor of its original discoverer [6]. In general, coupling to the environment will lead to other errors such as bit flips and decoherence, resulting in a mixed state at the end of the transport process. These kinds of errors, however, are correctable (after restoring energy degeneracy to the qubit basis $\{|0\rangle,|1\rangle\}$ if necessary) by using standard entanglement purification techniques [7]. The phase error in Eq. (7), however, is inextricably mixed with the synchronization offset between Alice and Bob, as we will argue below, and it cannot be purified asynchronously.

Although Ψ_δ is still a dark state under time evolution, it no longer has the key property of invariance under arbitrary unitary transformations $\hat{U}\otimes\hat{U}$. In particular, an equivalent form in terms of entangled clock states [such as in Eq. (5)] is not available for Ψ_δ [8]. Instead,

$$\Psi_\delta=\left(\frac{1+e^{i\delta}}{2\sqrt{2}}\right)(|-\rangle_A|+\rangle_B-|+\rangle_A|-\rangle_B) + \left(\frac{1-e^{i\delta}}{2\sqrt{2}}\right)(|+\rangle_A|+\rangle_B-|-\rangle_A|-\rangle_B), \tag{8}$$

and a measurement by Bob or Alice in the $\{|+\rangle,|-\rangle\}$ basis will leave the other party's clock in a superposition of clock states $|+\rangle$ and $|-\rangle$, which, if Bob and Alice were to follow the above QCS protocol blindly, effectively introduces an (unknown) synchronization offset of $-\delta/\Omega$ between them.

III. QCS AS TELEPORTATION OF CLOCKS

This connection between δ and the time-synchronization offset is much easier to understand by adopting a different point of view for the QCS protocol: one which is based on teleportation [3]. Accordingly, the essence of the QCS protocol can be viewed as the teleportation of clock states between Bob and Alice using the singlet states Ψ (or, in the present case, the imperfect singlets Ψ_δ). More explicitly, suppose Bob and Alice arrange, through prior classical communications, the teleportation of a known quantum state $\alpha|0\rangle_{B'}+\beta|1\rangle_{B'} \in\mathcal{H}_{B'}$ from Bob to Alice via the singlet Ψ_δ. Since the teleported state, as well as Bob's Bell-basis states [3]

$$\Psi^\pm\equiv\frac{1}{\sqrt{2}}(|0\rangle_B|1\rangle_{B'}\pm|1\rangle_B|0\rangle_{B'}),$$

$$\Phi^\pm\equiv\frac{1}{\sqrt{2}}(|0\rangle_B|0\rangle_{B'}\pm|1\rangle_B|1\rangle_{B'}) \tag{9}$$

are, in general, time dependent, the standard teleportation protocol needs to be slightly modified in the following way. The parties need to agree on a time, which we may take without loss of generality to be $t_B=0$ as measured by Bob's local clock, at which the following three actions will be performed instantaneously by Bob.

(i) Prepare an ancillary two-state system B' in the known quantum state $\alpha|0\rangle_{B'}+\beta|1\rangle_{B'}$, where α and β are complex numbers previously agreed on by the two parties.

(ii) Select a specific singlet Ψ_δ as in Eq. (7), and construct a Bell basis for $\mathcal{H}_B\otimes\mathcal{H}_{B'}$ that has the form Eq. (9) at $t_B=0$.

(iii) Perform a measurement in this basis and communicate its outcome to Alice through a classical channel. Upon receipt of this outcome, Alice is then to rotate the (collapsed) quantum state of her half of the singlet Ψ_δ (now a vector in the Hilbert space $\mathcal{H}_A$) by one of the four unitary operators

$$\hat{M}_{\Psi^\pm}=\begin{pmatrix}\pm1 & 0\\ 0 & -e^{-i\Omega t_A}\end{pmatrix},$$

$$\hat{M}_{\Phi^\pm}=\begin{pmatrix}-e^{-i\Omega t_A} & 0\\ 0 & \pm1\end{pmatrix}, \tag{10}$$

depending on whether the transmitted outcome of Bob's measurement is one of Ψ^+,Ψ^-,Φ^+ or Φ^-. Here t_A denotes Alice's proper time (as measured by her local clock) at the moment she performs her unitary rotation. Now let the (unknown) synchronization offset between Bob and Alice be τ, so that $t_B=t_A+\tau$. It is easy to show that the state teleported to Alice under this arrangement will have the form

$$\alpha|0\rangle_A+e^{i(-\Omega\tau+\delta)}\beta|1\rangle_A, \tag{11}$$

as obtained by Alice immediately following her unitary operation (one of $\hat{M}_{\Psi^\pm}$, $\hat{M}_{\Phi^\pm}$ [Eqs. (10)]) on $\mathcal{H}_A$.

A number of key results can now be easily read out from Eq. (11).

(1) If $\delta=0$, i.e., under the same assumption as in the original QCS protocol [1] that the shared singlet states are pure, the time-synchronization offset τ can be immediately determined by Alice (recall that α and β are known to both parties). Hence, the synchronization result of the QCS protocol can equivalently be achieved through teleportation.

(2) Conversely, if $\tau=0$, i.e., if Bob and Alice have their clocks synchronized to begin with, or if $\Omega=0$, i.e., if the qubits spanning the local Hilbert spaces $\mathcal{H}_A$ and $\mathcal{H}_B$ are degenerate, then δ can be immediately determined by Alice. Hence, purification of the phase-offset singlet Ψ_δ is possible under either of these two conditions.

(3) If, on the other hand, none of the quantities Ω,τ, and δ vanish, then the two unknowns τ and δ are inextricably mixed in the only phase observable $-\Omega\tau+\delta$, and asynchronous purification cannot be achieved via teleportation.

This last conclusion can be greatly clarified and strengthened by a Lorentz-invariant formulation of the above teleportation protocol (which, as we just argued, is equivalent to the original QCS), and it is this formulation we will turn to next.

IV. LORENTZ-INVARIANT ANALYSIS OF QCS

The key ingredient in any relativistic discussion of quantum-information theory is the space-time dependence of the qubit states. The "true" Hilbert space to which the quantum state of a singlet belongs is, accordingly, $L^2(\mathbf{R}^4)\otimes\mathcal{H}_A\otimes L^2(\mathbf{R}^4)\otimes\mathcal{H}_B$, where each $L^2(\mathbf{R}^4)$ is supposed to account for the space-time wave function of each two-state system in the entangled pair [for simplicity (but without any loss of generality), we consider only scalar (as opposed to spinor) qubits]. We will assume in what follows that background space-time is flat (Minkowski), and that the space-time dependence of each system's wave function can be approximated by that of a plane wave. In a more careful treatment, plane waves should be replaced by localized, normalizable wave packets.

We emphasize that a fully relativistic theory of quantum information would have to be formulated in the framework of relativistic quantum field theory. Since such a full-fledged formalism does not yet exist, we will confine our attention to a naive, "first-quantized" analysis, which is adequate for a qualitative understanding of the role of Lorentz invariance in QCS.

Denote the four-velocities of Alice and Bob by u_A and u_B, respectively, so that $u_A \cdot u_A = u_B \cdot u_B = -1$ [we will adopt the sign convention in which Minkowski metric on $\mathbf{R}^4$ has the form $\eta = -dt \otimes dt + dx \otimes dx + dy \otimes dy + dz \otimes dz$, and use the abbreviation $a \cdot b$ to denote $\eta(a,b)$ for any two four-vectors a and b]. The wave four-vectors of Alice's and Bob's atoms then have the form

$$k^0{}_J = m_0 u_J, \quad k^1{}_J = (m_0 + \Omega) u_J, \tag{12}$$

where m_0 is the ground-state rest mass of each (identical) two-level atom, and $k^0{}_J$ and $k^1{}_J$ denote the wave vectors corresponding to the ground and excited states of the atoms, respectively, where $J = A,B$. The plane-wave space-time dependence of the wave functions corresponding to the ground and excited states of each of the atoms can then be written in the form

$$|0\rangle_J \to e^{ik^0{}_J \cdot x}|0\rangle_J, \quad |1\rangle_J \to e^{ik^1{}_J \cdot x}|1\rangle_J, \tag{13}$$

where $J = A,B$, and x denote an arbitrary point (event) in space-time (a four-vector). Simple algebra then shows that, up to an overall phase factor (which can always be ignored), the wave function corresponding to the singlet state Eq. (7) can be expressed as a two-point space-time function of the form

$$\Psi_\delta(x_1,x_2) = |0\rangle_A|1\rangle_B - e^{i\Phi_\delta(x_1,x_2)}|1\rangle_A|0\rangle_B, \tag{14}$$

where x_1 and x_2 denote space-time points along the world lines of Alice and Bob, respectively, and $\Phi_\delta(x_1,x_2)$ is the Lorentz-invariant two-point phase function

$$\Phi_\delta(x_1,x_2) \equiv \Omega(u_A \cdot x_1 - u_B \cdot x_2) + \delta. \tag{15}$$

In the important special case where $u_A = u_B = u$, i.e., when Alice and Bob are comoving (and it makes sense to synchronize their clocks), Φ_δ takes the simpler form

$$\Phi_\delta(x_1,x_2) = \Omega u(x_1 - x_2) + \delta. \tag{16}$$

In the comoving case, Eq. (16), (when $u_A = u_B$), the singlet wave function $\Psi_\delta(x_1,x_2)$ is invariant under arbitrary Lorentz transformations including translations. This is in contrast with the general case, where the phase function $\Phi_\delta(x_1,x_2)$ [Eq. (15)] does not have translation invariance. This dependence on the choice of origin of coordinates is a manifestation of the fact that Ψ_δ is not a dark state unless $u_A = u_B$.

V. DISCUSSION: IS ASYNCHRONOUS ENTANGLEMENT PURIFICATION POSSIBLE?

The teleportation protocol of Sec. III (which is equivalent to the original QCS protocol of [1]) demonstrates that as long as x_1 and x_2 are timelike separated events in space time, the relative phase $\Phi_\delta(x_1,x_2)$ can be directly observed by Alice and Bob via quantum measurements followed by classical communication of the outcomes. An observation of $\Phi_\delta(x_1,x_2)$ would commence by the selection by Alice and Bob of space-time points x_1 and x_2 along their respective world lines at which they wish to measure this invariant phase function. Bob then would carry out his part of the teleportation protocol of Sec. III at his proper time corresponding to the event x_2, and broadcast the outcome to Alice along a nonspacelike communication path that reaches Alice before x_1. Alice would subsequently apply her unitary rotation [Eqs. (10)] sharp at her proper time corresponding to the event x_1. The resulting teleported state then has the form Eq. (11), where the relative phase is precisely $\Phi_\delta(x_1,x_2)$. Conversely, since the wave function contains all knowledge that can ever be obtained about a quantum system, the *only* (classical) observable associated with the singlet state Ψ_δ that contains any information about δ is $\Phi_\delta(x_1,x_2)$.

Focusing now on the comoving case $u_A = u_B$, the above fact implies that the phase offset δ *cannot* be observed in isolation; only the combination two-point function $\delta + \Omega u(x_1 - x_2)$ [Eq. (16)] is accessible to direct measurement. On the other hand, clock synchronization between Bob and Alice is equivalent to identification of pairs of events $(x_1^{(i)},x_2^{(i)})$ such that $u(x_1^{(i)} - x_2^{(i)}) = 0$. Therefore, by making a sequence of measurements of the relative phase function $\Phi_\delta(x_1,x_2)$, Alice and Bob can use the singlets Ψ_δ as a shared quantum-information resource to (i) synchronize their clocks if $\delta = 0$, and (ii) measure and purify δ if they have synchronized clocks to start with. In the general case of an unknown δ and an unknown time-synchronization offset, however, δ by itself is not observable, and, consequently, Ψ_δ cannot be purified without first establishing time synchrony between the two parties.

Note that this argument is *completely independent* of the particular protocol that may be used to purify the entanglement Eq. (7). Instead, the argument relies entirely on the nature of the space-time wave function describing an entangled pair, and this "universality" is its primary significance. The crucial observation is that the invariant two-point phase function $\Phi_\delta(x_1,x_2)$ is the *only* observable in the singlet state Eq. (7), and this function depends not only on the *a priori* relative phase δ, but also, through its dependence, on *both* x_1 and x_2, on the *a priori* time-synchrony information between Alice and Bob. Since the relative-phase information cannot be separated from time-synchrony information as long as the qubits remain nondegenerate, no protocol that does not rely on prior time synchrony in an essential way can purify the entanglement so as to distill pure ($\delta = 0$) entangled pairs.

VI. CONCLUSIONS AND FUTURE WORK

By using entangled (nondegenerate) qubits as a resource shared between spatially separated observers, the QCS protocol as reformulated above in Secs. III–V allows the direct measurement of certain nonlocal, covariant phase functions on space-time. Moreover, this functionality of the protocol is straightforward to generalize to many-particle entanglement [9]. While these results give hints of a profound connection between quantum information and space-time structure, they fall just short of providing a practical clock-synchronization algorithm because of the uncontrolled phase offsets [e.g, δ in Eq. (7)] that arise inevitably during the distribution of entanglement. Since, as we showed above, these phase offsets cannot be purified asynchronously after they are already in place, a successful completion of the (singlet based) QCS algorithm would need some method of entanglement distribution that avoids the accumulation of relative phase offsets. We believe a complete clock-synchronization algorithm based on quantum information theory will likely result from one of the following approaches.

"Phase-locked" entanglement distribution. It may be possible to use the inherent nonlocal (Bell) correlations of the singlet states (which remain untapped in the current QCS protocol), and implement a "quantum feedback loop," which, during entanglement transport, will help keep the phase offset δ vanishing to within a small tolerance of error. For example, states of the form

$$\frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_{A'}|1\rangle_B|0\rangle_{B'}-|1\rangle_A|0\rangle_{A'}|0\rangle_B|1\rangle_{B'}), \quad (17)$$

where two pairs of atoms (the primed and the unprimed pair) are entangled together, are not only dark but also immune to phase offsets during transport of the pairs to Alice and Bob (provided both pairs are transported along a common world line through the same external environment). Can such phase-offset-free states be used to control the purity of singlets during transport?

Entanglement distribution without transport. Physically moving each prior-entangled subsystem to its separate spatial location is not the only way to distribute entanglement. An intriguing idea, recently discussed by Cabrillo *et al.* [10], proposes preparing two spatially separated atoms in their long-lived excited states $|1\rangle_A|1\rangle_B$. A single-photon detector, which cannot (even in principle) distinguish the direction from which a detected photon arrives, is placed halfway between the atoms. When one of the atoms makes a transition to its ground state, and the detector registers the emitted photon, the result of its measurement is to put the combined two-atom system into the entangled state

$$\frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B+e^{i\phi}|1\rangle_A|0\rangle_B), \quad (18)$$

where ϕ is a random phase. Is there a similar procedure (based on quantum measurements rather than physical transport) that creates entanglement with a controlled rather than random phase offset ϕ?

Another method of entanglement distribution without transport, recently investigated in detail by Haroche and coworkers [11] is (in very rough outline) the following: Start with a single-mode cavity whose excitation frequency is tuned to Ω. Send the pair of atoms A and B into the cavity one after the other, with atom B first. Initially, both atoms and the cavity are in their ground states

$$|0\rangle_A\otimes|0\rangle_B\otimes|0\rangle_{EM}, \quad (19)$$

where $|0\rangle_{EM}$ denotes the vacuum state of the cavity. After atom B is in the cavity, apply a $\pi/2$ pulse on it, which transforms the state Eq. (19) into

$$\frac{1}{\sqrt{2}}|0\rangle_A\otimes(|0\rangle_B\otimes|1\rangle_{EM}-|1\rangle_B\otimes|0\rangle_{EM}). \quad (20)$$

When both atoms are in the cavity, apply a second, π pulse, this time on the atom A, thereby transforming the state Eq. (20) into

$$\frac{1}{\sqrt{2}}(|1\rangle_A\otimes|0\rangle_B-|0\rangle_A\otimes|1\rangle_B)\otimes|0\rangle_{EM}, \quad (21)$$

which, for the atom pair A and B, is in the desired form Eq. (2) up to an overall phase factor. Since at each step the overall quantum state (of atoms and the electromagnetic field) is dark, no relative phase errors can creep in, and pure entanglement distribution is achieved between atoms A and B. Can this method be adapted to design a practical entanglement transfer protocol between distant pairs of atoms using a controlled cavity environment?

Avoiding entanglement distribution altogether. Can classical techniques of clock synchronization be improved in accuracy and noise performance by combining them with techniques from quantum-information theory, which do not necessarily involve (nondegenerate) entanglement distribution? A recent proposal in this direction was made by Chuang in [12].

After this paper was submitted for publication, further ideas utilizing quantum entanglement *without* entanglement distribution to improve the accuracy of classical Einstein synchronization have been proposed in Refs. [13–15] (see also [16] for an overview).

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[2] There is one additional complication in the QCS protocol, having to do with making sure that the Hadamard transforms [Eqs. (3)] performed by Alice and Bob are phase matched. This matching is equivalent to setting up a canonical isomorphism between the Hilbert spaces $\mathcal{H}_A$ and $\mathcal{H}_B$, and is relatively straightforward to implement as discussed in detail in [1]. Throughout the rest of this paper, we will assume without further comment that such an isomorphism has been set up by Alice and Bob prior to the operations under discussion; Ref. [1] should be consulted for further details.

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[8] If the phase offset δ were known, such an equivalent entangled-clock expression for Ψ_δ would be available in terms of modified clock states $|\pm_\delta\rangle_A \equiv (|0\rangle_A \pm e^{i\delta/2}|1\rangle_A)/\sqrt{2}$ and $|\pm_\delta\rangle_B \equiv (|0\rangle_B \pm e^{-i\delta/2}|1\rangle_B)/\sqrt{2}$; but if δ is unknown, Alice and Bob can instead simply purify Ψ_δ by agreeing to rotate Bob's $|0\rangle$ basis state by $e^{i\delta}$.

[9] For example, to states of the form

$$\Psi_N = \frac{1}{\sqrt{N!}} \sum_\sigma (-1)^\sigma |0\rangle_{A\sigma(1)} |1\rangle_{A\sigma(2)} \cdots |N-1\rangle_{A\sigma(N)},$$

where $A1, A2, \ldots, AN$ denote N observers who have, distributed to them, N identical atoms with N distinct internal energy levels, and the sum σ is over all permutations of $\{1,2,\ldots,N\}$. The state Ψ_N is a generalization of the singlet Ψ [Eq. (2)] in that it is invariant under $U \otimes U \otimes \cdots \otimes U$ for arbitrary unitary U.

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